IS U(1)$_H$ A GOOD FAMILY SYMMETRY?

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ABSTRACT

We analyze U(1)$_H$ as a horizontal symmetry and its possibilities to explain the known elementary-fermion masses. We find that only two candidates, in the context of SU(3)$_c \otimes$SU(2)$_L \otimes$U(1)$_Y \otimes$U(1)$_H$ nonsupersymmetric, are able to fit the experimental result m$_b \ll$m$_t$.

1 INTRODUCTION

The pattern of fermion masses, their mixing, and the family replication, remain as the most outstanding problems of nowadays particle physics. The successful standard model (SM) based on the local gauge group $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$ can tolerate, but not explain the experimental results. Two main features that a consistent family theory should provide are:

(i)-Within each charge sector, the masses increase with generation by large factors:

$$m_u << m_c << m_t; \quad m_d << m_s << m_b; \quad m_e << m_\mu << m_\tau$$

(ii)-Even if one restricts to the heaviest family, the masses are still quite different:

$$m_\tau \sim m_b << m_t.$$

The horizontal survival hypothesis[1] was invented in order to accommodate (i), under the (wrong) assumption that $m_\tau \sim m_b \sim m_t$. The idea of radiative symmetry breaking in a supersymmetric extension of the SM[2] depends crucially on the existence of one quark with a mass comparable to the SM breaking scale, but it can not explain why this was the top quark instead of the bottom quark. The modified horizontal survival hypothesis[3] was introduced in order to explain the full extent of (i) and (ii), but a dynamically realization of this hypothesis is still lacking. Of course, these hypothesis and ideas rest on the assumption that all the dimension four Yukawa couplings in a well behaved theory should be of order one.

Related to (i) and (ii) is the fact that the Cabibbo-Kobayashi-Maskawa quark mixing matrix is near to the identity, but it is a common prejudice to assume that the appropriate family symmetry may explain this fact as a consequence of (i) and (ii). In what follows we will enlarge the SM gauge group with an extra $U(1)_H$ horizontal local gauge symmetry (the simplest multi-generational continuous symmetry we can think of). We then show that the structure $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_H$ by itself is able to explain (ii), and that the simplest supersymmetric (SUSY) extension of this model without a μ-term can not cope with (ii).

2 $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_H$ as an anomaly-free model

Our attempt is to keep the number of assumptions and parameters down to the minimum possible, and to try to construct a model which explains both features (i) and (ii) at the lowest possible energy scale. We therefore demand cancellation

of the triangular (chiral) anomalies[4] by the power counting method, including the mixed gravitational(grav) anomaly[5]. The alternative of cancelling the anomalies by a Green-Schwarz mechanism[6] has been already considered in Refs.[7], and corresponds to the construction of a model string-motivated which demands the inclusion of physics near the Plank scale.

$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_H$ as a continuous gauge group, with $U(1)_H$ as a family symmetry, was introduced long ago in Ref.[8], (revived recently in the context of SUSY string-motivated models in Refs.[7, 9, 10]). There are two different versions of the model, corresponding to two different ways of cancelling the chiral anomalies. One is demanding cancellation of the anomalies for each generation and the other one is cancelling the anomalies between generations.

2.1 Cancellation of anomalies in each generation

Assuming there are no right-handed neutrinos, using the $U(1)_Y$ and $U(1)_H$ charges displayed in Table (1), and demanding freedom from chiral anomalies for $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)_H$, we get:

$$[SU(2)_L]^2 U(1)_H \quad : \quad Y_{\psi_i} + 3Y_{\chi_i} = 0 \tag{1}$$

$$[SU(3)_c]^2 U(1)_H \quad : \quad 2Y_{\chi_i} + Y_{U_i} + Y_{D_i} = 0 \tag{2}$$

$$[U(1)_Y]^2 U(1)_H \quad : \quad 2Y_{\psi_i} + 4Y_{E_i} + \frac{2}{3}Y_{\chi_i} + \frac{16}{3}Y_{U_i} + \frac{4}{3}Y_{D_i} = 0 \tag{3}$$

$$U(1)_Y [U(1)_H]^2 \quad : \quad -Y_{\psi_i}^2 + Y_{E_i}^2 + Y_{\chi_i}^2 - 2Y_{U_i}^2 + Y_{D_i}^2 = 0 \tag{4}$$

$$[grav]^2 U(1)_H \quad : \quad 2Y_{\psi_i} + Y_{E_i} = 0 \tag{5}$$

$$[U(1)_H]^3 \quad : \quad 2Y_{\psi_i}^3 + Y_{E_i}^3 + 6Y_{\chi_i}^3 + 3Y_{U_i}^3 + 3Y_{D_i}^3 = 0. \tag{6}$$

The solution to Eqs.(1)-(6) is [8, 11]

$$Y_{H_\eta}^i = \alpha_i Y_{SM_\eta},$$

where α_i is an arbitrary number different for each generation, and Y_{SM_η} is the $U(1)_Y$ charge for the η multiplet.

These $U(1)_H$ charges cannot explain the feature (ii) which demands that at tree level only the top quark acquires a mass, and therefore that the Higgs field with $U(1)_H$ charge $Y_{H\phi}$ satisfies:

$$Y_{\chi 3} + Y_{U_3} = Y_{H\phi}$$

$$Y_{\chi_3} + Y_{D_3} \neq -Y_{H\phi} = Y_{H\phi^\star}.$$

But once the first of these equations is satisfied, Eq. (2) above implies $Y_{\chi_3} + Y_{D_3} = -Y_{H\phi}$. Therefore, if a top quark mass arises at tree level ($Y_{H\phi} = \alpha_3$), a bottom mass arises as well at the same level.

Adding right-handed neutrinos $N^c{}_{i,L}$ to our set of fundamental fields does not change this conclusion since Eq.(2) stays valid [the only changes are in Eqs. (5) and (6) which are now replaced by

$$\begin{aligned}
[\text{grav}]^2 U(1)_H \quad &: \quad 2Y_{\psi_i} + Y_{E_i} + Y_{N_i} = 0 \\
[U(1)_H]^3 \quad &: \quad 2Y_{\psi_i}^3 + Y_{E_i}^3 + 6Y_{\chi_i}^3 + 3Y_{U_i}^3 + 3Y_{D_i}^3 + Y_{N_i}^3 = 0].
\end{aligned} \tag{7}$$

2.2 Cancellation of anomalies between generations

If the $U(1)_H$ anomalies are cancelled by an interplay among generations, Eqs. $(1) -$ (6) should be understood with a summ over $i = 1, 2, 3$. Eq. (4) then reads

$$\sum_i (-Y_{\psi_i}^2 + Y_{E_i}^2 + Y_{\chi_i}^2 - 2Y_{U_i}^2 + Y_{D_i}^2) = 0. \tag{8}$$

Obviously a solution to the new anomaly constraint equations which are linear or cubic in the Y_{η_i} is

$$\sum_{i=1}^{3} Y_\eta^i = 0$$

for each η. We will limit ourselves to this type of solutions and within this set we will consider only those for which the ψ_i and U_i H-hypercharges are fixed to satisfy either

$$Y_{\psi_1} = \delta_1 \equiv \delta, \, Y_{\psi_2} = \delta_2 = -\delta, \, Y_{\psi_3} = \delta_3 = 0,$$

$$Y_{U_1} = \delta_1' \equiv \delta', \, Y_{U_2} = \delta_2' = -\delta', \, Y_{U_3} = \delta_3' = 0,$$

or any set of relations obtained from the former equations by a permutation of the indices $i = 1, 2, 3$. The solutions can then be divided onto four classes according to the way the cancellations occur in Eq.(8).

CLASS A

$Y_{E_i} = Y_{\psi_i} = \delta_i$ and $Y_{D_i} = Y_{\chi_i} = Y_{U_i} = \delta_i'; i = 1, 2, 3$. A model with a tree-level

top quark mass arises if $Y_{H_\phi} = Y_{\chi_i} + Y_{U_j}$ for some i and j. There are five different models in this class characterized by $Y_{H_\phi} = \pm 2\delta', \pm \delta'$ and 0 respectively. Any of this five models becomes nonviable if it gives rise to a tree-level bottom mass. That is if there exists a k and a l for wich $Y_{\chi_k} + Y_{D_l} = -Y_{H_\phi}$. For example, if $Y_{H_\phi} = 2\delta'$ then $i = j = 1$ and $k = l = 2$ satisfy the previous equations; this is signaled in Table (2) by the entry $(1,1)_U; (2,2)_D$ in the Class A column and the $2\delta'$ row. The fact that in Table (2) there is at least one D-type entry for every U-type one for all the five models of Class A, means that none of them is viable. This fact can be easily understood by noticing that $Y_{\chi_i} + Y_{U_j}$ changes sign under the interchange $1 \leftrightarrow 2$ in the i,j indices, and that in Class A $Y_{D_i} = Y_{U_i}$. Therefore, for a fixed Y_{H_ϕ}, $Y_{\chi_i} + Y_{U_j} = -(Y_{\chi_k} + Y_{D_l})$.

CLASS B

$Y_{\chi_i} = Y_{\psi_i} = \delta_i$ and $Y_{D_i} = Y_{E_i} = Y_{U_i} = \delta'_i; i = 1, 2, 3$. There are nine different models in this class characterized by $Y_{H_\phi} = (\delta \pm \delta'), -(\delta \pm \delta'), \pm \delta, \pm \delta'$ and 0 respectively. Since again $Y_{D_i} = Y_{U_i}$ none of those models is viable.

CLASS C

$Y_{D_i} = Y_{\psi_i} = \delta_i$ and $Y_{\chi_i} = Y_{E_i} = Y_{U_i} = \delta'_i; i = 1, 2, 3$. There are now eleven different models in this class characterized by $Y_{H_\phi} = \pm 2\delta', (\delta \pm \delta'), -(\delta \pm \delta'), \pm \delta, \pm \delta'$ and 0. As can be seen from Table (2) for $\delta \neq \pm \delta', \pm 2\delta', \pm 3\delta'$ and $\delta' \neq 0$, there are two models in which only one U-type mass and none D-type one develops at tree-level. These models are:

Mark I. For a Higgs field with $(\text{U}(1)_Y, \text{U}(1)_H)$ hypercharges $(-1, 2\delta')$.

Mark II. For a Higgs field with $(\text{U}(1)_Y, \text{U}(1)_H)$ hypercharges $(-1, -2\delta')$.

The rest of the models in this classd are non-viable because a tree-level bottom mass arises in them.

CLASS D

This is a special class which is a particular case of Classes A,B, and C, for which $\delta = \delta'$, which in turn implies $Y_{E_i} = Y_{\psi_I} = Y_{D_i} = Y_{\chi_i} = Y_{U_i}$. As far as the quark mass spectrum is concerned this class is equivalent to class A.

Two comments: first, in Ref.[8], the class of solutions A, B, and C were all lumped together in class D, which in turn forbids the two models classified as Mark I and Mark II above. Second, adding right-handed neutrino fields does not change our analysis at all, either by setting $Y_{N_1} = -Y_{N_2} = \delta, Y_{N_3} = 0$ (or permutations of the indices 1,2,3); or by imposing $Y_{N_i} = 0, i = 1, 2, 3$ in order to implement the seesaw mechanism[12].

3   SU(3)$_c$⊗SU(2)$_L$⊗U(1)$_Y$⊗U(1)$_H$ SUSY

For the supersymmetric extension of the standard model (SSM) new particles of spin 1/2 are introduced which are the supersymmetric partners of the Higgs fields and gauge bosons. These higgsinos and gauginos do not contribute to the U(1)$_Y$ anomaly because they are chosen vector-like with respect to the quantum numbers of the SM, in such a way that the SM relationship Q$_{EM}$=T$_{3L}$+Y/2 holds, which in turn implies the U(1)$_Y$ charges in Table (3). As in the minimal SSM two different supermultiplets of Higgs fields ϕ_D and ϕ_U are introduced.

Now, the simplest way to have a gauge symmetry U(1)$_H$ anomaly-free when the new spin 1/2 members of the supermultipletes are included, is to demand that these new fermions are vector-like with respect to U(1)$_H$. That is, to impose $Y_{\eta_s} = \kappa Y_{SM_\eta}$, with κ an arbitrary constant. For $\kappa = \alpha_3$, ϕ_U and ϕ_D will produce tree level masses for the top and bottom quark respectively. This particular solution is not consistent with (ii).

But is there other solution to the anomaly constraint equations which is consistent with (ii)? Let us see:

If all the electrically neutral gauginos are allowed to have Majorana masses, then $Y_\gamma = Y_{\gamma'} = Y_g = Y_{(W,B)}$=0. Now, the higgsinos $\widetilde{\phi}_U$ and $\widetilde{\phi}_D$ do not carry a generational index, but if they are to produce masses at least for the third generation, then their charges have to be related to the charges of the third family (see the third paper in Ref.[7]). If this is the case then the anomaly cancellation equations are Eqs. (1)-(6) for $i = 1, 2$, but for $i = 3$ they are:

$$[\text{SU}(2)_L]^2 U(1)_H \quad : \quad Y_{\psi_3} + 3Y_{\chi_3} + Y_{\phi_U} + Y_{\phi_D} = 0 \tag{9}$$

$$[\text{SU}(3)_c]^2 U(1)_H \quad : \quad 2Y_{\chi_3} + Y_{U_3} + Y_{D_3} = 0 \tag{10}$$

$$[U(1)_Y]^2 U(1)_H \quad : \quad 4Y_{E_3} + \frac{2}{3}Y_{\chi_3} + \frac{16}{3}Y_{U_3} + \frac{4}{3}Y_{D_3} + 2(Y_{\psi_3} + Y_{\phi_U} + Y_{\phi_D}) = 0 \tag{11}$$

$$U(1)_Y[U(1)_H]^2 \quad : \quad -Y_{\psi_3}^2 + Y_{E_3}^2 + Y_{\chi_3}^2 - 2Y_{U_3}^2 + Y_{D_3}^2 + Y_{\phi_U}^2 - Y_{\phi_D}^2 = 0 \tag{12}$$

$$[\text{grav}]^2 U(1)_H \quad : \quad 2Y_{\psi_3} + Y_{E_3} + 2Y_{\phi_U} + 2Y_{\phi_D} = 0 \tag{13}$$

$$[U(1)_H]^3 \quad : \quad 2Y_{\psi_3}^3 + Y_{E_3}^3 + 6Y_{\chi_3}^3 + 3Y_{U_3}^3 + 3Y_{D_3}^3 + 2Y_{\phi_U}^3 + 2Y_{\phi_D}^3 = 0 \tag{14}$$

There are two solutions to these equations. The first one is $Y_{\chi_3} = Y_{E_3}/6 = -Y_{U_3}/4 = Y_{D_3}/2 = -Y_{\phi_D}/3 = -Y_{\psi_3}/3$ and $Y_{\phi_U} = -Y_{\phi_D}$. The second one is $Y_{\chi_3} = Y_{E_3}/6 = -Y_{U_3}/4 = Y_{D_3}/2 = -Y_{\phi_D}/3$ and $Y_{\psi_3} = -Y_{\phi_U}$. For the first solution masses for the Up and Down sector are generated simultaneously, and for the second solution

$Y_{U_3} + Y_{\chi 3} \neq -Y_{\phi_U}$, failing to give a mass for the Up sector.

So for this extension of the SSM, the $U(1)_H$ anomalies can not vanish simultaneously with the generation of only a tree-level mass for the top quark. The alternative is to go to higher mass scales and cancell the $U(1)_H$ anomalies by a Green-Schwarz mechanism[6]. Then, for SUSY to work there must be a μ-term, meaning that the hypercharges of the Higgs fields can be changed[10]. But this analysis has allready been carried through in the literature[7, 9, 10].

4   ACKNOWLEDGMENTS

$U(1)_Y$ and $U(1)_H$ charges for the known fermions. i=1,2,3 is a flavor index related to the first, second and third generations. The Y_{SM} values stated are family independent.

	$\psi_{i,L}=(N_i,E_i)_L$	$E_{i,L}^c$	$\chi_{i,L}=(U_i,D_i)_L$	$U_{i,L}^c$	$D_{i,L}^c$	$N_{i,L}^c$
Y_{SM}	-1	2	$1/3$	$-4/3$	$2/3$	0
$Y_{H_\eta}^i$	Y_{ψ_i}	Y_{E_i}	Y_{χ_i}	Y_{U_i}	Y_{D_i}	Y_{N_i}

Table(2)

Summary of three-level mass term for all the possible models for the local gauge group $SU(3)_c\otimes SU(2)_L\otimes U(1)_Y\otimes U(1)_H$. A Higgs field with a hypercharge Y_{H_ϕ} different to the ones in the first column does not produce a mass term in the quark sector.

Y_{H_ϕ}	CLASS A	CLASS B	CLASS C
$2\delta'$	$(1,1)_U;(2,2)_D$		$(1,1)_U$
$-2\delta'$	$(2,2)_U;(1,1)_D$		$(2,2)_U$
0	$(1,2)_U;(2,1)_U;(3,3)_U;(1,2)_D;(2,1)_D;(3,3)_D$	$(3,3)_U;(3,3)_D$	$(1,2)_U;(2,1)_U;(3,3)_U;(3,3)_D$
δ'	$(1,3)_U;(3,1)_U;(2,3)_D;(3,2)_D$	$(3,1)_U;(3,2)_D$	$(1,3)_U;(3,1)_U;(2,3)_D$
$-\delta'$	$(2,3)_U;(3,2)_U;(1,3)_D;(3,1)_D$	$(3,2)_U;(3,1)_D$	$(2,3)_U;(3,2)_U;(1,3)_D$
$\delta+\delta'$		$(1,1)_U;(2,2)_D$	$(2,2)_D$
$-\delta+\delta'$		$(2,1)_U;(1,2)_D$	$(2,1)_D$
$\delta-\delta'$		$(1,2)_U;(2,1)_D$	$(1,2)_D$
$-\delta-\delta'$		$(2,2)_U;(1,1)_D$	$(1,1)_D$
δ		$(1,3)_U;(2,3)_D$	$(3,1)_D$
$-\delta$		$(2,3)_U;(1,3)_D$	$(3,2)_D$

$U(1)_Y$ and $U(1)_H$ charges for the sparticles of spin 1/2. $\widetilde{\gamma}$ and $\widetilde{\gamma}'$ are the gauginos related to $U(1)_Y$ and $U(1)_H$ respectively, $\tilde{g}$ stand for the eight gluinos, etc.

	$\widetilde{\phi}_U$	$\widetilde{\phi}_D$	$\widetilde{\gamma}$	$\widetilde{\gamma}'$	$\tilde{g}$	$(\tilde{W},\tilde{B})$
Y_{SM}	1	-1	0	0	0	0
Y_{η_s}	Y_{ϕ_U}	Y_{ϕ_D}	Y_γ	$Y_{\gamma'}$	Y_g	$Y_{(W,B)}$

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